TORQUE GENERATION IN WIND TURBINES

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INTRODUCTION

Wind turbines extract power from asymmetries in the wind stream by converting it into mechanical rotational energy of its rotors. This energy is transmitted to the loads such as a transmission or gearbox or an electrical generator through rotating shafts. These must be adequately designed to operate under normal and gusty wind conditions. They must be lubricated, cooled, properly aligned and free of vibrations.

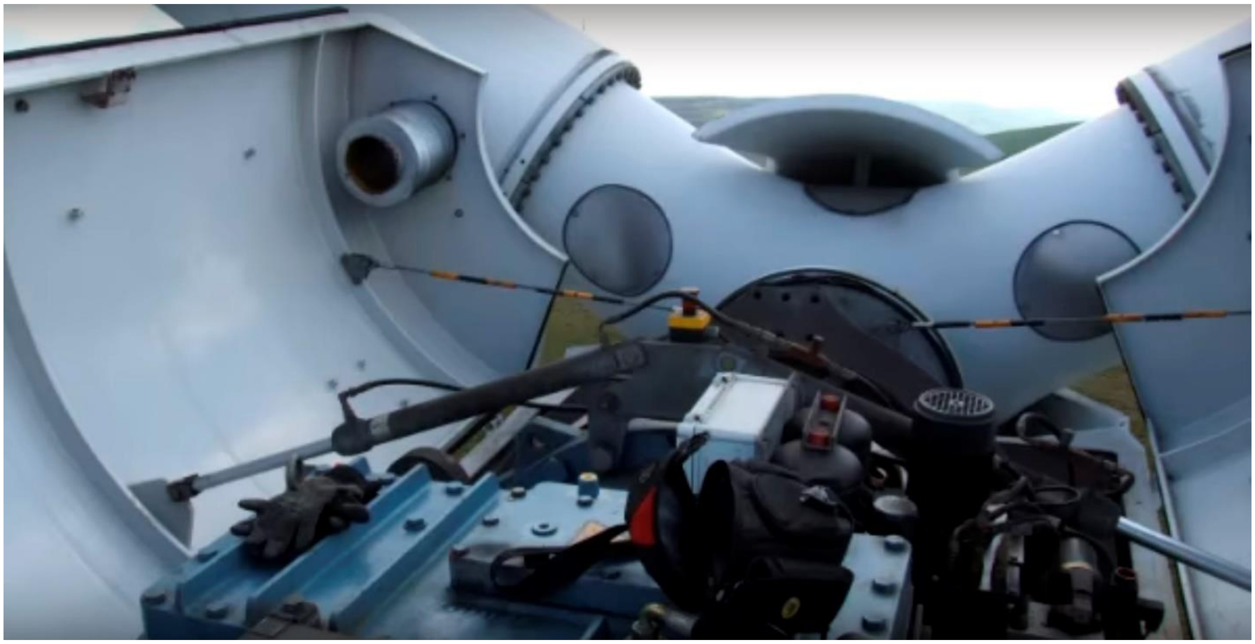

Figure 1. Wind turbine nacelle showing hub and gearbox.

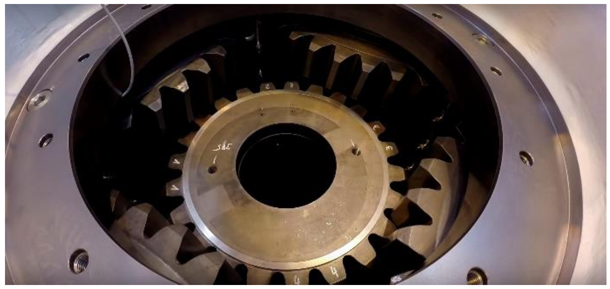

Figure 2. Wind turbine planetary transmissions during manufacture.

Figure 3. Assembled Winenergy 8 MW wind turbine gearbox.

SHAFT OPERATION

When a shaft is operated at an angular rotational speed ω, it is associated with the power P being transmitted through a torque T according to:

$$P = \omega.T\ [\frac{\text{radian}}{\text{sec}}.\frac{\text{Newton.meter}}{\text{radian}}],[\frac{\text{Joule}}{\text{sec}}],[\text{Watts}] \qquad (1)$$

This defines the torque as:

$$T = \frac{P}{\omega}\,[\frac{\text{Watt.sec}}{\text{radian}}],[\frac{\text{Joule}}{\text{radian}}],[\frac{\text{Newton.meter}}{\text{radian}}] \qquad (2)$$

This definition of the torque applies to a rotating shaft, and is different from the torque on a stationary structure such as a structural tower which would have units of Newton.meter.

STRESS GENERATION

As torque is applied to a rotating shaft, internal stresses or pressures are applied on the shaft material. Since this stress tends to shear instead of stretching or compressing the shaft, it is designated as a shear stress. The shear stress in a solid shaft is a function of the radial position r on the shaft axis and is largest at the shaft's surface. In terms of the polar moment of inertia J of the shaft it is given by:

$$\sigma_s(r) = \frac{T}{J} r [\frac{\text{Newton.meter.radian}}{\text{radian.m}^4}\text{.meter}],[\frac{\text{Newton}}{\text{m}^2}],\text{Pascal},\text{Pa} \qquad (3)$$

where the polar moment of inertia of a solid shaft with radius r_0 is given by:

$$J = \frac{1}{2}\pi r_0^4 [\frac{\text{m}^4}{\text{radian}}] \qquad (4)$$

MAXIMUM SHEARING STRESS

A rotating shaft must be designed so as to carry a given torque T by determining the maximum shearing stress that can be allowed for a chosen shaft material. The maximum stress occurs at the surface of the shaft when $r = r_0$. The shaft radius that will carry this maximum stress from Eqns. 3, 4, is given by:

$$\sigma_s(r_0) = \frac{T}{J} r_0 = \frac{T}{\frac{1}{2}\pi r_0^4} r_0 = \frac{2T}{\pi r_0^3}$$

$$r_0^3 = \frac{2T}{\pi \sigma_s(r_0)}$$

From which:

$$r_0 = \sqrt[3]{\frac{2T}{\pi\sigma_s(r_0)}}\text{[meter]} \qquad (5)$$

Significant safety and ignorance factors need to be introduced at that stage.

TRANSMISSION, GEARBOX DESIGN

Consider the design of a wind generator with an electrical output of:

$$P_e = 0.5 MWe$$

Accounting for the generator efficiency, the power at the transmission output would be:

$$P_t = \frac{P_e}{\eta_g} \qquad (6)$$

For a generator efficiency of 90 percent, this would be:

$$P_t = \frac{500,000}{0.90} = 555,555\ Watts$$

And the power at the transmission input would be:

$$P_m = \frac{P_t}{\eta_t} = \frac{P_e}{\eta_g \eta_t} \qquad (7)$$

For a transmission efficiency of 90 percent, this would be:

$$P_m = \frac{500,000}{0.9x0.9} = 617,284\ Watts$$

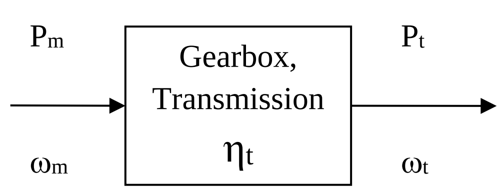

Figure 4. Power and rotational speeds across a transmission or gear box.

Taking the rotational speed of the generator at 1,200 rpm, yields:

$$\omega_t = 2\pi \frac{1,200}{60} = 40\pi [\frac{\text{radians}}{\text{sec}}]$$

Taking the rotational speed of the rotor shaft as 24 rpm, corresponding to a gearing ratio of:

$$Gearing\ ratio : GR = \frac{1,200}{24} = 50$$

yields:

$$\omega_m = 2\pi \frac{24}{60} = \frac{4}{5}\pi [\frac{\text{radians}}{\text{sec}}]$$

From Eqn. 2 the torques at the high speed and low speed shafts torques become:

$$T_t = \frac{P_t}{\omega_t} = \frac{555,555}{40\pi} = 4,421 [\frac{\text{N.m}}{\text{rad}}]$$

$$T_m = \frac{P_m}{\omega_m} = \frac{617,284}{\frac{4}{5}\pi} = 245,610 [\frac{\text{N.m}}{\text{rad}}]$$

A maximum stress for steel shafts is recommended as 55 Mpa. Accounting for a factor of safety FS of 3 and an ignorance factor IF of 2 yields for the design maximum stress:

$$\sigma_{s,\max}(r_0) = \frac{\sigma_s(r_0)}{FS.IF} = \frac{\sigma_s(r_0)}{3x2} = \frac{\sigma_s(r_0)}{6}$$

$$\sigma_{s,\max}(r_0) = \frac{55}{6} = 9.2 MPa$$

Substituting in Eqn. 5 yields for the high speed and low speed shaft radii:

$$r_{0,t} = \sqrt[3]{\frac{2T_t}{\pi\sigma_{s,\max}(r_0)}} = \sqrt[3]{\frac{2\times 4,421}{\pi\times 9.2x10^6}} = \left[305.9\times10^{-6}\right]^{1/3} = 6.738\times10^{-2}[\text{meter}] = 6.7 \text{ cm}$$

$$r_{0,m} = \sqrt[3]{\frac{2T_m}{\pi\sigma_{s,\max}(r_0)}} = \sqrt[3]{\frac{2\times 245,610}{\pi\times 9.2x10^6}} = \left[16,995.8\times10^{-6}\right]^{1/3} = 25.709\times10^{-2}[\text{meter}]=25.7 \text{ cm}$$

Notice the difference in the radii of the high speed shaft (6.7 cm) and the low speed shaft (25.7 cm). Because of its size and weight, the low speed shaft must be kept at a minimum length in wind turbines designs.

GEARING RATIO OF SIMPLE GEARS

For a simple set of gears, the radial speeds at the contact interface point for the driver and driven gear should be equal:

$$v_{driver} = v_{driven}$$
$$\omega_{driver} r_{driver} = \omega_{driven} r_{driven}$$

From which we can deduce the ratios foo the angular speed, the radii and the number of teeth:

$$i = \frac{\omega_{driver}}{\omega_{driven}} = \frac{r_{driven}}{r_{driver}} = \frac{z_{driven}}{z_{driver}}$$

For two gears, the Gearing Ratio is defined as:

$$Gearing\ ratio : GR = i = \frac{z_{driven}}{z_{driver}} = \frac{number\ of\ teeth\ of\ driven\ gear\ 2}{number\ of\ teeth\ of\ drive\ gear\ 1}$$

PLANETARY EPICYCLICAL SUN PLANETS GEARS

Figure 5. Planetary epicyclical gear sun planets.

Figure 6. Planetary gears terminology.

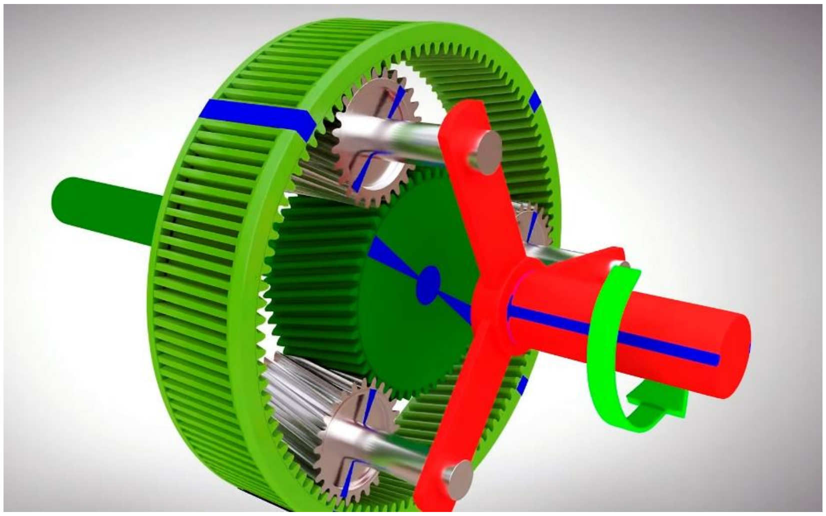

Figure 7. Planetary epicyclical gear sun/planets/ring/carrier.

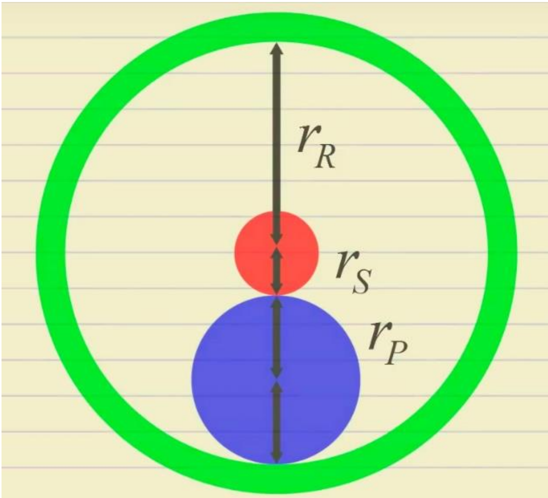

Figure 8. Planetary gear sun/planet/ring radii.

The following relationships apply between the sun, planets and ring radii:

$$r_{Ring} = r_{Sun} + 2r_{Planet}$$
$$2r_{Planet} = r_{Ring} - r_{Sun}$$

The speeds of the carrier and the sun gear and the carrier can be written as:

$$v_{Carrier} = \omega_{Carrier}(r_{Sun} + r_{Planet})$$
$$v_{Sun} = \omega_{Sun} r_{Sun}$$

Choosing:

$$v_{Sun} = 2v_{Carrier}$$

Leads to:

$$\omega_{Sun} r_{Sun} = 2\omega_{Carrier}(r_{Sun} + r_{Planet})$$

$$\frac{\omega_{Sun}}{\omega_{Carrier}} = \frac{2(r_{Sun} + r_{Planet})}{r_{Sun}} = \frac{2r_{Sun} + 2r_{Planet}}{r_{Sun}}$$

$$= \frac{2r_{Sun} + r_{Ring} - r_{Sun}}{r_{Sun}} = \frac{r_{Sun} + r_{Ring}}{r_{Sun}}$$

$$= 1 + \frac{r_{Ring}}{r_{Sun}}$$

The Gearing Ratio in this case between the sun gear and the carrier can be derived as:

$$i = \frac{\omega_{Sun}}{\omega_{Carrier}} = 1 + \frac{r_{Ring}}{r_{Sun}} = 1 + \frac{z_{Ring}}{z_{Sun}}$$

GEAR BOX MACHINING AND HARDENING

Figure 9. Gear box machining.

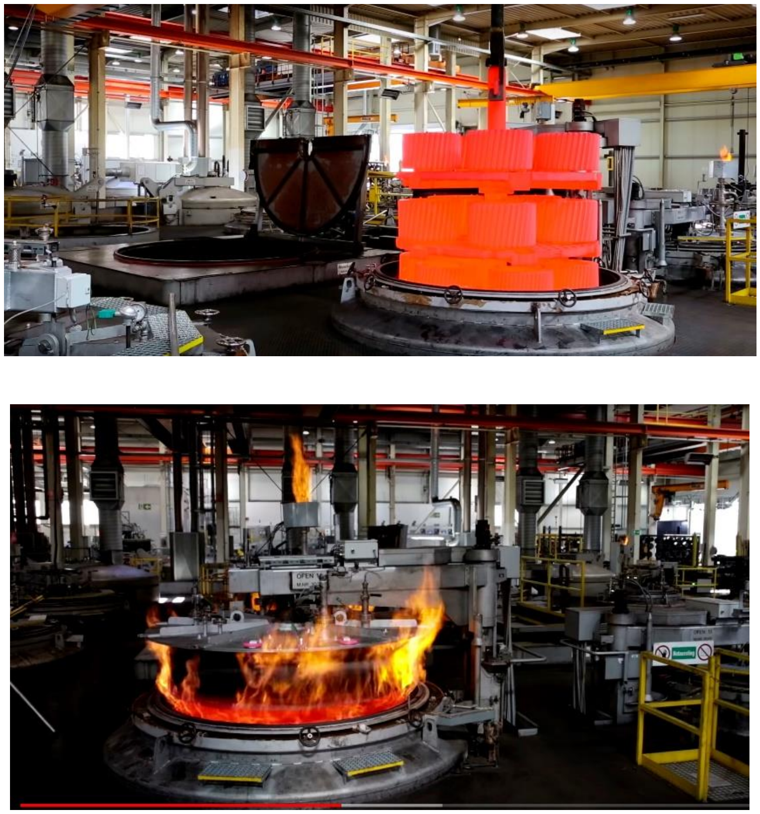

Figure 10. Gear box hardening heat treatment.

TRANSMISSION OVERHALL

Figure 11. Damage to nine year old transmission showing abrasion and pitting.

Figure 12. Overhaul of nine years old planetary transmission.

EXAMPLE

$$z_S = 12, z_R = 60,$$
$$i = 1 + \frac{60}{12} = 1 + 5 = 6$$